



# On Analog Power Consumption

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## Outline

- Introduction
- Basics of analog power consumption
- Comparison analog and digital
- RF elements
- Example RF frontend
- Example ADC
- Conclusions

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## Introduction

Analog power consumption is complex and badly understood

Text books and practice specialize on performance constraints rather than power

This is an attempt to address analog design for low power and to define lower bounds to analog power consumption

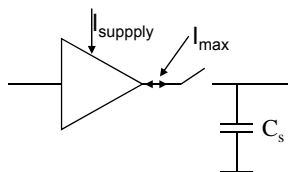
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## Basics of analog power consumption

### Ideal sampler



$$\text{Noise voltage generated: } v_{nS}^2 = \frac{kT}{C_s}$$

Maximum sine voltage with supply voltage  $V_{FS}$ :

$$v_s^2 = \frac{V_{FS}^2}{8} \quad \text{Gives dynamic range: } DR = \frac{v_s^2}{v_{nS}^2} = \frac{V_{FS}^2 C_s}{8kT}$$

$$\text{or, capacitance needed: } C_s = \frac{8kTDR}{V_{FS}^2}$$

To drive the capacitor we need a current of  $I = C_s V_{FS} 2f_s$ ,

leading to a power consumption of  $P_S = IV_{FS} = 16kTf_s DR$

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## Basics of analog power consumption

### Ideal sampler

$$P_S = IV_{FS} = 16kTf_sDR$$

This expression is valid for any switched capacitor circuit  
(Example a first order switched capacitor filter)  
First described by Vittoz 1990.

Note that this expression is *independent of technology*

Note that this expression is *independent of supply voltage*  
(but assumed  $V_{FS}=V_{dd}$ ;  $V_{dd}$  is supply voltage)

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## Basics of analog power consumption

### Ideal sampler - impact of technology

In the above derivation we used  $C_s = \frac{8kTDR}{V_{FS}^2}$

But  $C_s$  can not be made smaller than the smallest capacitance of the actual technology,  $C_{min}$ .

So actually we should use  $C = \max(C_s, C_{min})$  in power estimation.

In the following we use the input capacitance of a minimum inverter as  $C_{min}$ .  $C_{min} = 4 * C_{gn}$ .  $C_{gn}$  will scale with technology as the feature size (half feature size - half capacitance)

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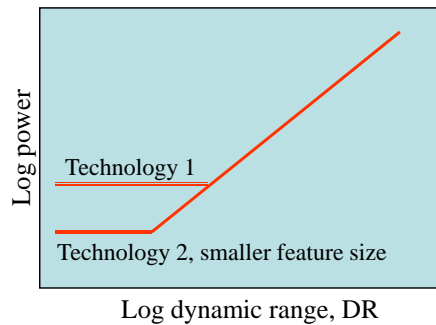
## Basics of analog power consumption

### Ideal sampler - impact of technology

Large DR - independent technology

Smaller DR - technology dependent  
Smaller feature size - less power

Note, scaled as (feature size)<sup>3</sup>  
( $P_S \sim V_{FS}^2 C_{min}$ )



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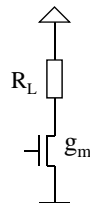


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## Basics of analog power consumption

### The transistor



Drain noise current:  $i_{nD}^2 = 4kT\gamma g_m B$

Equivalent input noise:  $v_{nG}^2 = 4kT\gamma \frac{1}{g_m} B$

Dynamic range:  $DR = \frac{V_{FS}^2}{8} \frac{g_m}{4kT\gamma B}$

Required  $g_m$  for dynamic range DR:  $g_m = \frac{32kT\gamma B}{V_{FS}^2} DR$

Required drain current:  $I_D = g_m V_{eff}$

Power consumption:  $P = IV_{FS} = 32kT\gamma B \frac{V_{eff}}{V_{FS}} DR$  (compare  $P_S = 16ktf_s DR$ )

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## Basics of analog power consumption

### The transistor – a note on $V_{eff}$ and $\gamma$

$V_{eff}$  definition:  $V_{eff} = I_D / g_m$        $\gamma$  defined from noise measurements

Bipolar transistor:  $V_{eff} = kT/q$        $\gamma = 1/2$  (shot noise)

Long channel MOST:  $V_{eff} = (V_G - V_T)/2$        $\gamma = 2/3$  (resistance noise)

Long channel MOST in subthreshold:  $V_{eff} = m kT/q$ ,  $m = 1.2 \dots 1.5$

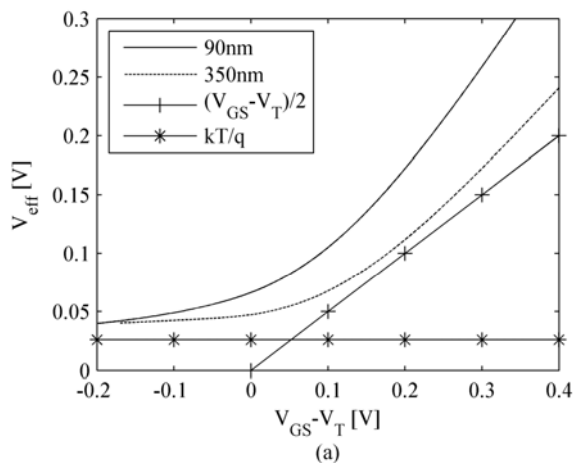
Submicron MOST:  $V_{eff}$  in transition region       $\gamma \sim 1.5-2$

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## Basics of analog power consumption



$V_{eff}$  vs  $V_G$  for  
a 90nm process  
a 350 nm process

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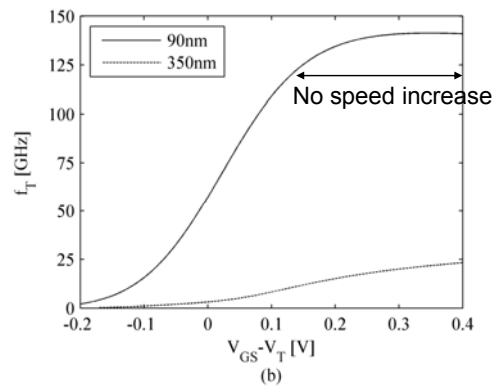


## Basics of analog power consumption

### The transistor – a note on $V_{eff}$ and speed.

Speed controlled by  $f_T$

$$f_T = g_m / 2\pi C_g$$



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## Basics of analog power consumption

### The transistor – a note on $V_{eff}$ and input swing

For a linear transistor behaviour we expect a limited drain current variation, say  $I_{DC} \pm \Delta I_d = I_{DC} \pm I_{DC}/2$ , with  $I_{DC} = g_m V_{eff}$ .

We then have  $\Delta V_g = \Delta I_d / g_m = V_{eff} / 2$ , or  $V_{FSin} = 2\Delta V_g = V_{eff}$

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## Basics of analog power consumption

### The transistor – a note on $V_{dd}$

We assumed supply voltage =  $V_{FS}$

With higher supply,  $V_{dd}$ , we define voltage efficiency  $\eta_v = V_{FS}/V_{dd}$

Then power *increases* by  $1/\eta_v$

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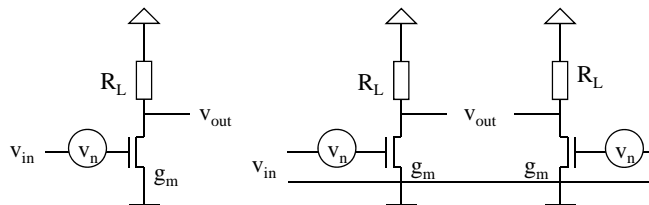


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## Basics of analog power consumption

### The transistor – a note on differential circuits



Same transistors, same bias, same signal per transistor

$$DR_{diff} = \frac{(2v_{in})^2}{2v_n^2} = 2DR_{SE} \quad P_{diff} = 2P_{SE}$$

Same dynamic range  
Same power

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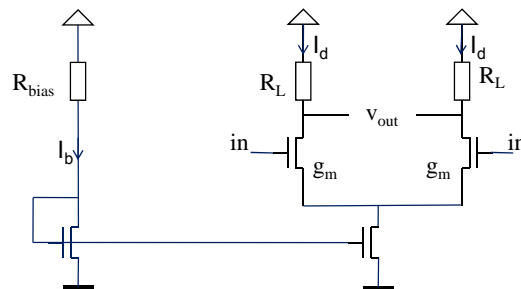


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## Basics of analog power consumption

### The transistor – a note on bias circuits



Current efficiency,  $\eta_I = 2I_d / (I_b + 2I_d)$ . Note,  $I_b$  may need to be large for eg. low noise

Note – bias circuit may be shared

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## Basics of analog power consumption

### The transistor – impact of technology

#### Regarding minimum $g_m$

We can always reduce  $g_m$  to adapt to DR by reducing  $V_G$ .

BUT this leads to small  $V_{eff}$ , which may limit speed or input swing

Maybe we will have  $g_{mmin}$  and  $I_{DCmin}$ .

#### Power proportional to $V_{eff}/V_{FS}$

Classical CMOS, this relation was kept constant

$V_{FS}$  ( $V_{dd}$ ),  $V_G$  and  $V_T$  scaled with process scaling

Submicron CMOS,  $V_{eff}$  is reduced very slowly.

So power may *increase* with reduced supply voltage! (Annema et. al.)

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## Basics of analog power consumption

### The transistor – a conclusion

A transistor amplifier (single ended or differential) uses power:

$$P = IV_{FS} = 32kT\gamma B \frac{V_{eff}}{V_{FS}} DR$$

This is constrained by:

Increased by  $1/\eta_v$  for  $V_{FS} < V_{dd}$

Increased by  $1/\eta_i$  for extra currents (eg. bias)

$V_{eff}$  must be large enough for transistor speed ( $f_T = g_m / 2\pi C_g$  large enough)

$V_{eff}$  must exceed input swing,  $V_{FSin}$

Technology dependence at low DR - complex

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## Basics of analog power consumption

### A switched C amplifier

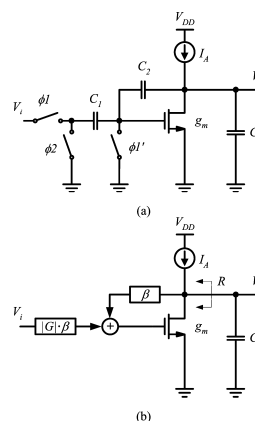
Input data sampled during  $\Phi 1$   
Output data generated (and sampled by following stage) during  $\Phi 2$

Nominal gain,  $G = -C_1/C_2$

Return factor  $\beta = C_2/(C_1 + C_2)$

Output resistance  $R = 1/\beta g_m = (1 + |G|)/g_m$

Output capacitance,  $C_{LA} = C_1 C_2 / (C_1 + C_2) + C_3$



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## Basics of analog power consumption

### A switched C amplifier

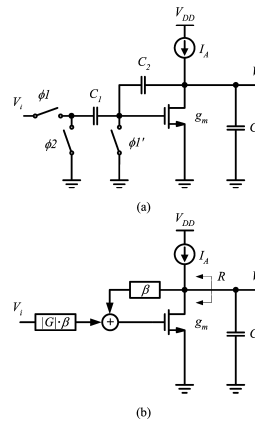
Output noise voltage:

$$\overline{v_o^2} = 4kT\gamma g_m R^2 \cdot \frac{1}{4RC_{LA}} = \frac{kT}{C_{LA}} \cdot \gamma \cdot (1+|G|).$$

Requiring a dynamic range of  $DR = V_{FS}^2/v_o^2$ :

$$C_{LA} = 8kT\gamma(1+|G|) \frac{DR}{V_{FS}^2}.$$

This gives a requirement on capacitance (similar to  $kT/C$  requirement)



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## Basics of analog power consumption

### A switched C amplifier

We have further a speed requirement

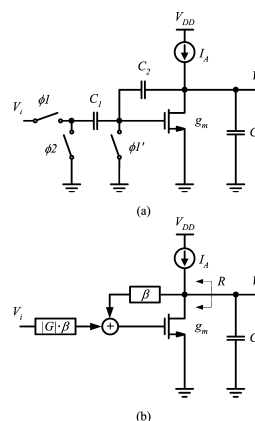
Time constant of circuit is  $\tau = RC = (1+|G|)C_{LA}/g_m$

$\tau$  must be small enough for settling to an accuracy of  $1/\sqrt{DR}$  within  $T_{se}$ :

$$\tau = \frac{2T_{se}}{\ln(DR)}$$

This gives a requirement on  $g_m$  and therefore on  $I_{DC}$

$$I_{DC} = \frac{C_{LA}V_{eff}(1+|G|)\ln(DR)}{2T_{se}}$$



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## Basics of analog power consumption

### A switched C amplifier

We may refine the speed requirement by also considering slewing. With  $I_{DC}$  we get a slewing time of:

$$T_{sl} = \frac{C_{LA} V_{FS}}{I_{DC}}$$

So, totally we need  $T_{se} + T_{sl} = 1/2f_s$  for slewing+settling leading to  $I_{DC}$ . From this we estimate total power:

$$P_{DC} = I_{DC} V_{dd} = \frac{I_{DC} V_{FS}}{\eta_v} = \left( 1 + \frac{V_{eff} (1 + |G|) \ln(DR)}{2V_{FS}} \right) \frac{2C_{LA} V_{FS}^2 f_s}{\eta_v}$$

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## Basics of analog power consumption

### A switched C amplifier

$$P_{DC} = I_{DC} V_{dd} = \frac{I_{DC} V_{FS}}{\eta_v} = \left( 1 + \frac{V_{eff} (1 + |G|) \ln(DR)}{2V_{FS}} \right) \frac{2C_{LA} V_{FS}^2 f_s}{\eta_v}$$

Note that the slewing term corresponds to the "ideal" sampling power

In this case we may allow  $V_{eff} < V_{FS}$ ; if transistor is shut off, there is slewing

Note that again power is proportional to the capacitance,  $C_{LA}$

Capacitance controlled by dynamic range (noise level) and more...

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## Basics of analog power consumption

### A switched C amplifier – impact of technology

For small DR  $C_{LA_n}$  may be smaller than  $C_{min}$ . Then it must be replaced by  $C_{min}$ .

$$C_{LA} = \max(C_{LA_n}, C_{min})$$

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## Basics of analog power consumption

### A switched C amplifier – impact of mismatch

Sometimes we need to consider accuracy because of mismatch. Eg. we need to reproduce G within a certain accuracy, say  $\Delta G/G$ , corresponding to, say, a relative variance of G,  $\sigma_G$ ,  $3\sigma_G = \Delta G/G$ .

As G depends on two capacitors, we may estimate  $\sigma_G = \sqrt{2}\sigma_C$ , where  $\sigma_C$  is the process spread of capacitances. With capacitor area A we have:

$$\sigma_C = \frac{K_\sigma}{\sqrt{A}} \quad C = K_C A$$

with typical  $K_\sigma = 1\%$  and  $K_C = 1\text{fF}/\mu\text{m}^2$  (90...350nm processes). Thus we need the capacitor to be larger than

$$C_{LA_m} = \frac{2K_C K_\sigma^2}{\sigma_G^2} = \frac{2K_C K_\sigma^2}{9} \left(\frac{\Delta G}{G}\right)^{-2}$$

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## Basics of analog power consumption

### A switched C amplifier – impact of mismatch

An interesting observation:  $C_{LAn}$  proportional to  $DR/V_{FS}^2$   
 $C_{LAm}$  proportional to  $DR$

Traditionally mismatch limitations are considered worse than noise limitations

With our formulas we find that their effects will be similar around  $V_{FS}=1V$   
 So, mismatch less important for deep submicron processes?

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## Basics of analog power consumption

### A switched C amplifier – design scheme

1. **Decide on capacitance,  $C_{LA}=\max(C_{LAn}, C_{min}, C_{LAm})$**   
 Controlled by DR, technology, and accuracy
2. **Decide on speed; calculate  $g_m$**
3. **Decide on  $V_{eff}$**   
 Depends on speed and swing, leads to  $I_{DC}$
4. **Finish design, estimate power.**

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## Basics of digital power consumption

### Comparison to analog

Consider a single pole filter / 1-tap FIR filter

Analog, use transistor formula:  $P_a = 24kT\gamma \frac{V_{eff}}{V_{FS}} f_s 2^{2n}$   
(with  $DR=3 \cdot 2^{2n}/2$ ;  $B=f_s/2$ )

Digital. Need 1 m-coefficient, n-bit multiplier plus 1 n-bit adder  
Multiplier uses m adders; 1 adder use n FAs (full adders); each FA  
uses equivalent 9 inverters, ie.  $C=9C_{min}$ . Totally  $9(m+1)nC_{min}$ ,  
With  $m=6$  we have  $63nC_{min}$ .

$$P_d = \frac{1}{2} \alpha f_c 63nC_{min} V_{dd}^2$$

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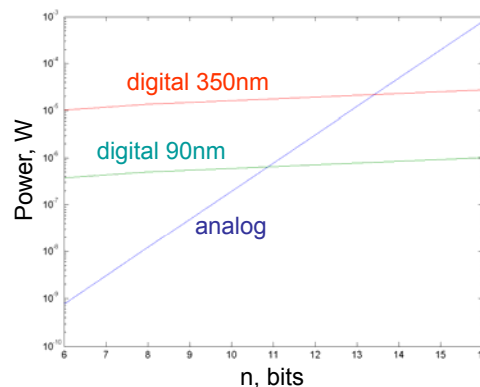
## Basics of digital power consumption

### Comparison to analog

$f_s=20\text{MHz}$   
 $\gamma=1.5$   
 $V_{eff}=0.1V_{FS}$

90nm:  $V_{dd}=1\text{V}$   
 $C_{min}=1\text{fF}$   
 $\alpha=0.1$

350nm  $V_{dd}=3\text{V}$   
 $C_{min}=3\text{fF}$   
 $\alpha=0.1$



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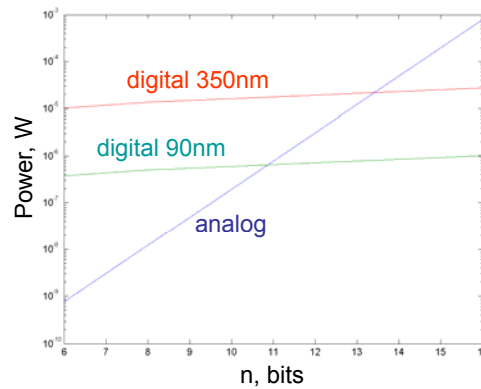
## Basics of digital power consumption

### Comparison to analog

Analog preferred for low DR

Digital preferred for large DR

Digital improved by scaling



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## RF elements

### The LNA

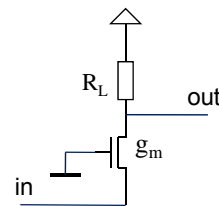
Normally we need wideband input matching and low noise figure. A good compromise is the common gate stage.

Matching:  $R_{in} = 1/g_m = R_s$  ( $R_s$  - source resistance)

Noise figure:  $NF = 1 + \gamma$

Power consumption:  $P = I_D V_{dd} = g_m V_{eff} V_{dd} = V_{eff} V_{dd} / R_s$

given by  $R_s$  (normally  $50\Omega$ ) and  $V_{eff}$  (chosen as small as allowed from speed and input swing requirement). Typically  $I_D = 2mA$



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## RF elements

### The LNA – impedance transformation

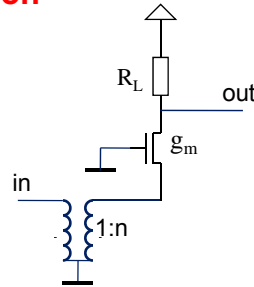
With a transformer, we may transform the source impedance to  $n^2 R_s$  thus reducing  $g_m$  and power consumption by  $n^2$ .

Limitations:

For large impedances, induced gate noise will impact – optimum impedance

Transformers not good at high frequencies  
- need tuning elements, narrow bandwidth

Transformers replaced by impedance transforming matching network  
- need tuning elements, narrow bandwidth



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## RF elements

### The LNA – current reuse, high linearity

Matching:  $R_{in} = 1/(g_{m1} + g_{m2}) = R_s$   
( $R_s$  - source resistance)

Noise figure:  $NF = 1 + \gamma$

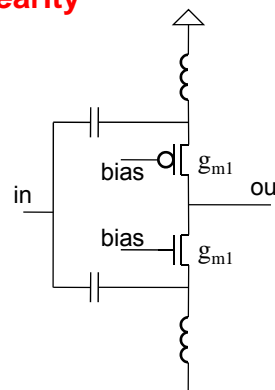
Power consumption ( $g_{m1} = g_{m2}$ ,  $V_{eff}$  same):

$$P = I_D V_{dd} = g_{m1} V_{eff} V_{dd} = V_{eff} V_{dd} / 2R_s$$

Half power, same supply current used twice.

Inverter-like structure, can be very linear  
(nonlinearities of n- and p-transistors cancel)

Note, for large input swing, large  $V_{eff}$



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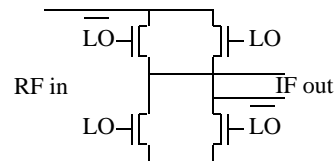
## RF elements

### Mixers - passive mixer

A passive mixer has zero power consumption

Also, can have reasonably good noise figure

Also, has very low 1/f noise



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## RF elements

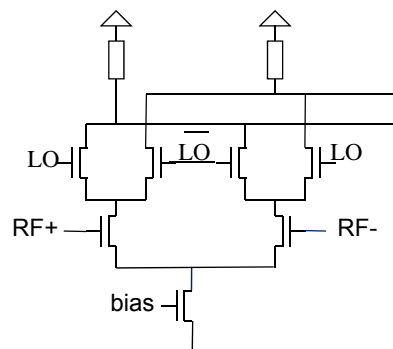
### Mixers - active mixer

An active mixer has gain and use power

Similar to LNA design.

Choose  $g_m$  for noise (and/or gain)

Adjust  $V_{eff}$  to speed or  $V_{FSin}$



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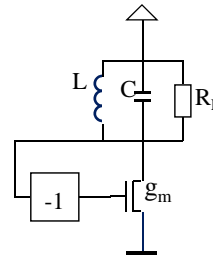
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## RF elements

### VCOs

The VCO, voltage controlled oscillator, is central in a radio. Key performance factor is phase noise.

We use a simple oscillator model, signal and noise is generated by the transistor noise current (The "-1" block can be a transformer or another transistor stage)



$$\text{Output voltage: } v_o^2 = \left| \frac{Z_L}{1 - g_m Z_L} \right|^2 i_n^2 = \frac{R_L^2 \omega_0^2}{4Q^2 \Delta \omega^2} i_n^2$$

## RF elements

### VCOs

$$\text{Noise current: } i_n^2 = 4kT\gamma g_m$$

$$\text{Output power: } P_o = \frac{V_{FS}^2}{8R_L}$$

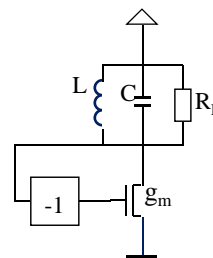
Relative noise spectral density:

$$S(\omega) = \frac{v_o^2}{R_L P_o} = \frac{\gamma k T \omega_0^2}{Q^2 P_o \Delta \omega^2}$$

$$\text{Half is phase noise: } L(\omega) = \frac{\gamma k T \omega_0^2}{2Q^2 P_o \Delta \omega^2}$$

Stored energy

Thermal energy



## RF elements

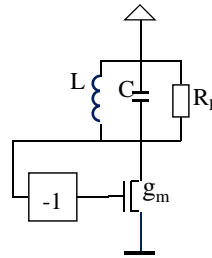
### VCOs

Power consumption,  $P_{DC} = I_{DC} V_{dd}$   
 $I_{DC} = g_m V_{eff}$ ,  $g_m = 1/R_L$ ,  $V_{eff} = V_{FS}$ ,  $V_{dd} = V_{FS}/\eta_v$

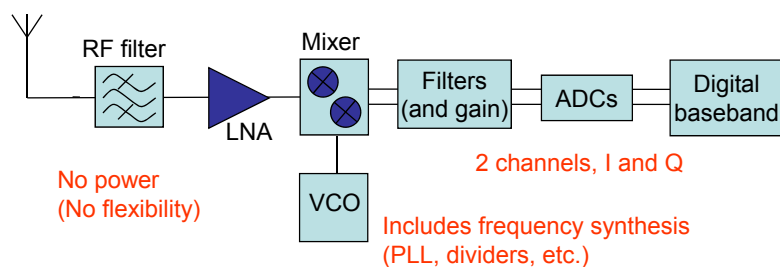
We can then estimate the power consumption:

$$P_{DC} = \frac{4\gamma kT}{\eta_v Q^2 L(\omega) \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

Typical requirement  $L(\omega) = -100 \text{ dBc/Hz}$  at  $\Delta\omega = 100 \text{ kHz}$  and  $\omega_0 = 1 \text{ GHz}$ ,  
 with  $\eta_v = 0.5$ ,  $\gamma = 1.5$ ,  $Q = 5$  (loaded Q) we have  $P_{DC} = 1.2 \text{ mW}$



## Example RF receiver frontend





## Example RF receiver frontend

	Requirements	Current	Real example
LNA	$R_s=50\Omega$ , $NF=4\text{dB}$	2mA	7-12mA
Mixer	Active	(2X2mA)	2X5-12mA
VCO	$L(\omega)=100\text{dBc/Hz @ } 100\text{kHz}$ 1.8-6GHz	3-36mA	4-10mA
IF	4th order; $BW=0.35\text{-}20\text{MHz}$	2X9.2mA	2X10mA

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## Example RF receiver frontend

Real case: IMEC receiver (Craninckx, et.al., ISSCC 2007), 1.2V supply

Assumptions:

Blocker level -18dBm (corresponds to  $IIP3=-9\text{dBm}$ )



4th order IF filter created by a 4 stage amplifier

In addition to the above blocks, Real case includes VGA, LO buffer, PLL divider and phase detector, increasing minimum power from 41 to 65mA

**In conclusion, a low power receiver design is close to our estimations**

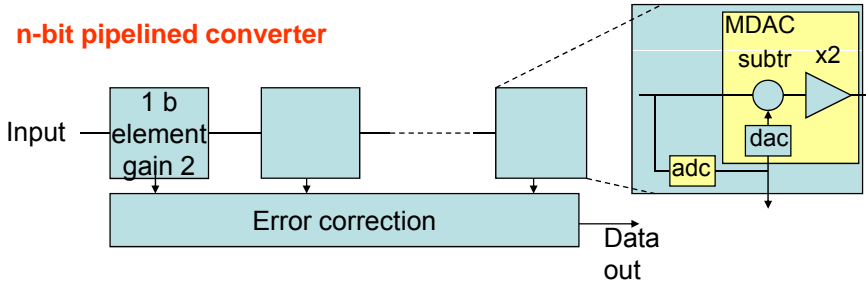
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## Example ADC



**n-bit pipelined converter**



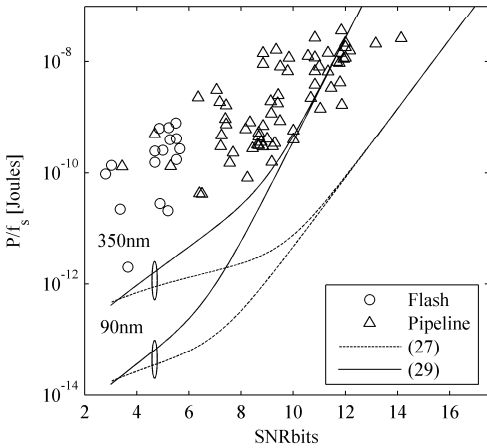
By introducing redundancy in each element (eg. 2b adc/dac) it is possible to remove offsets and correct gain errors. Then we do not need to consider accuracy.

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## Example ADC



Plot of minimum ADC power for two processes

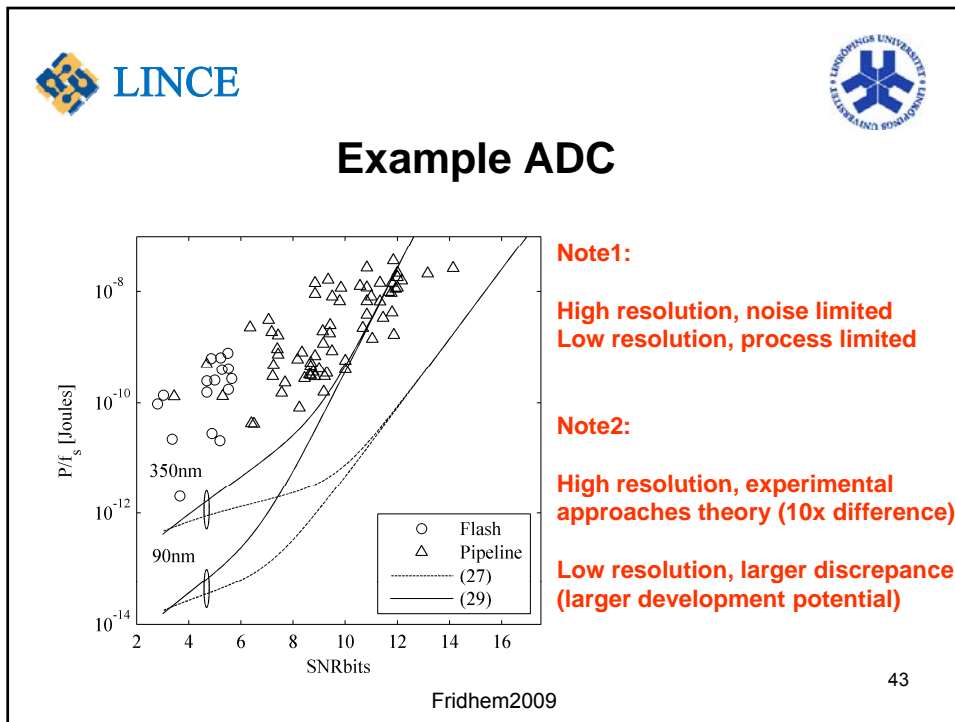


“(27)” this theory pipelined ADC  
“(29)”, theory flash ADC

Two processes, 90nm and 350nm  
( $V_{dd}=1V$ ,  $V_{eff}=100mV$ ,  $C_{min}=1fF$ ,  
 $V_{dd}=3V$ ,  $V_{eff}=300mV$ ,  $C_{min}=3fF$ )

Triangles, experimental pipelined  
Circles, experimental flash

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### Example ADC

**Commercial examples**

LTC2209, 16b, 160MS/s 1.45W

Our formula: 0.6W **VERY** close (very well developed technology)

ADC083000, 8b, 3GS/s, 1.8W

Our formula (pipe): 0.13mW, much to improve!

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## Conclusions

**Power consumption in analog parts can be addressed**

**Smart choices of architecture, circuit topology, and parameters can save power**

**For large dynamic range, digital often use less power**

More so in scaled technologies

Move functions to digital

Perform digital correction of analog parts to save analog power

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